**23BMT451**

**UG PROGRAM (4 YEARS HONORS) WITH SINGLE MAJOR**

**AT THE END OF FOURTH SEMESTER**

**MATHEMATICS-RING THEORY & PROBLEM SOLVING SESSIONS (Minor)**

**(w.e.f. Admitted Batch 2023-24)**

**Time: 3 Hours** **Maximum: 70 Marks**

## Section–A

**Answer any Five Questions.** **5x4=20**

1. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
2. Show that the intersection of two subrings is a subring.
3. Give an example of a ring which is not a principal ideal ring.
4. Show that the homomorphic image of commutative ring is commutative.
5. Find the factors of $x^4 + 4$ in $Z_5[x]$
6. Show that the characteristic of a Boolean ring is 2.
7. If U is an ideal of the ring R and a, b $\in R$ then prove that a+U = b+U $\Leftrightarrow a - b \in U$
8. If f is a homomorphism from a ring R into a ring S then Ker f is an ideal of R.

## Section– B

**Answer All Questions.** **5x10=50**

9. a) If p is a prime then prove that $Z_p$, the ring of integers modulo p, is a field.

**(Or)**

b) Show that a commutative ring R with unity is a field if and only if (0) and R are the only ideals of R.

10. a) Show that the necessary and sufficient condition for a nonempty subset S of a ring R to be a subring of R are i) $\forall a, b \in S \Longrightarrow a - b \in S$, ii) $\forall a, b \in S \Longrightarrow ab \in S$

**(Or)**

b) Show that the union of two ideals of a ring R, is an ideal of R if and only if one is contained in the other.

11. a) Show that the ring of integers is a principal ideal ring.

**(Or)**

b) Let S be an ideal of a ring R. Then show that
i) If R is commutative then R/S is commutative
ii) If R contains unity then R/S contains unity.

12. a) State and prove fundamental theorem of homomorphism of rings

**(Or)**

b) Show that an ideal S of a commutative ring R with unity is maximal if and only if the residue class ring R/S is a field.

13. a) State and prove Division Algorithm theorem in polynomials

**(Or)**

b) If F is a field, prove that every ideal in F[x] is a principal ideal.